Studia Geotechnica et Mechanica, Vol. XXIX, No. 3–4, 2007

ONE-DIMENSIONAL CONSOLIDATION OF POROELASTIC MEDIUM WITH RHEOLOGICAL SKELETON

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Abstract: In this paper, the author presents the results of numerical investigations of poroelastic medium behaviour (consisting of a skeleton and the fluid flowing through the voids in this skeleton), called Biot's body. It is assumed that the stress–strain state does not reach the failure limits of the body.

The starting point for these considerations is the phenomenon of one-dimensional consolidation of such a medium described by the fundamental poroelastic Biot's equation and completed by the Biot's–Darcy flow equation. Additionally in this description the rheological properties of the skeleton of Biots body are modelled by linear viscoelastic Zener's body (standard body).

Streszczenie: Przedstawiono wyniki badań numerycznych dotyczących zachowania się ośrodka porosprężystego (składającego się ze szkieletu i cieczy wypełniającej pory tego szkieletu), zwanego ciałem Biota, w zakresie stanu naprężenia, w którym nie występuje jeszcze utrata wytrzymałości ciała.

Punktem wyjścia rozważań jest zjawisko konsolidacji takiego ośrodka opisane równaniem porosprężystości Biota, uzupełnionym równaniem przepływu Biota–Darcy'ego. W opisie uwzględniono cechy reologiczne szkieletu ciała Biota, zakładając, że odpowiadają one zachowaniu modelu liniowo lepkosprężystego Zenera (standardowego). Założono bowiem, że podczas procesu konsolidacji ruch cząstek stałych może odbywać się po kontakcie: ciało stałe–ciało stałe bądź po warstewce wody związanej na powierzchni ciała stałego. Otrzymujemy tu zatem połączenie dwóch niezależnych procesów: i) konsolidacji związanej z zagęszczaniem ośrodka, które jest spowodowane odpływem filtracyjnym wody swobodnej, oraz ii) procesu pełzania rozumianego jako ruch fazy stałej ośrodka, któremu przeciwstawia się opór lepki wynikający z założenia, że przemieszczające się cząstki gruntu „ślizgają się" po warstewce wody związanej na powierzchni tych cząstek.

Резюме: Представлены результаты численных исследований, касающихся поведения пористо-упругой среды (состоящей из скелета и жидкости, заполняющей поры этого скелета), званной телом Биота, в пределах состояния напряжения, в котором еще не выступает потеря прочности тела.

В качестве исходной точки рассуждений выступает явление консолидации такой среды, описанное уравнением пористо-упругости Биота, пополненным уравнением течения Биота–Дарси. В описании учтены реологические свойства скелета тела Биота при предположении, что они отвечают поведению линейно вязко-упругой модели Зенера (стандартной). Было предположено, что во время процесса консолидации движение постоянных частиц может происходить по контакте: твердое тело–твердое тело или по слое воды, связанной на поверхности твердого тела. Здесь получено соединение двух независимых процессов: i) консолидации, связанной с уплотнением среды, которое вызвано фильтрационным отливом свободной воды, а также ii) процесса ползучести, понимаемой как движение постоянной фазы среды, которой противопоставлено вязкое сопротивление, вытекающее из предположения, что перемещивающиеся частицы почвы „скользят" по слое воды, связанной на поверхности этих частиц.

1. INTRODUCTION

This article presents an attempt to answer the following question: how do the viscosity properties (effects) of the soil matrix influence the stress–strain process of the two-phase poroelastic medium (water-saturated soil), called the Biot's body?

In order to find the solution, numerical investigations were conducted on one-dimensional consolidation of the Biot's body with Zener's rheological skeleton, which allows for viscosity of the solid phase. The dynamic viscosity as well as the volumetric viscosity were taken into consideration. The works of KISIEL et al. [5] and STRZELECKI et al. [8] have shown that in the case of cohesive soils, the assumption of viscosity properties of the skeleton is not unfounded. It is assumed that during the consolidation process the solid particles move on the contact between solid–solid, or in the majority of cases, on the water film bound to the surface of the solid. Hence we deal with a combination of two independent processes: i) consolidation connected with an increase in the density of a porous medium only due to the outflow of free water (according to KISIEL and LYSIK [6] this process corresponds to the Terzaghi's consolidation) and ii) creeping process regarded as the motion of the solid phase of the medium, opposed by the viscous resistance, resulting from the assumption that moving grains "are sliding" on the water film bound to its surface, STRZELECKI et al. [8].

As was mentioned earlier, the classical Biot's model does not consider the viscosity behaviour of the soil matrix, therefore Zener's body was additionally introduced. In the papers of KISIEL and LYSIK [6], KISIEL [4], KISIEL et al. [5], DERSKI [2], SAWICKI [7], many other more or less complicated rheological models can be found, for example: Kelvin's, Maxwell's, Goldsztejn's and Tan's models. Hence, section two of this paper gives an explanation of why the Zener's model is used. It also deals with the fundamental equations of consolidation, the rheological Zener's skeleton model, and the definition of numerical problem.

Section three contains the definition of the boundary value problems (classical Biot's model, Biot's model with Zener's skeleton) and the results. The discussion of the results and conclusions are presented in the fourth section.

2. BIOT'S MODEL WITH RHEOLOGICAL ZENER'S SKELETON

According to what has been already said in the introduction, the analysis of one-dimensional consolidation of soil medium is presented. The starting point for the considerations is the Biot's fundamental equation of poroelastisity; this equation and the Biot–Darcy's filtration law complement each other. One-dimensional equations of the inner equilibrium of a two-phase medium, omitting the body forces, are denoted by:

$$\begin{cases} N_2 \nabla^2 u_1 + \left(N_2 + A_2 - \dfrac{Q^2}{R} \right) \varepsilon_{,1} = -\dfrac{H}{R} \sigma_{,1}, \\ \dfrac{k}{f^2} \nabla^2 \sigma = \dfrac{1}{R} \dot{\sigma} - \dfrac{H}{R} \dot{\varepsilon}, \end{cases} \tag{1}$$

where $Q = H - R$, N_2, A_2, H, R are Biot's constants (BIOT [1], STRZELECKI et al. [8]), k is Darcy's filtration coefficient and f stands for medium porosity. This model assigns elasticity properties to the soil skeleton (through the parameters N, A) and to the water that fills its pores (through the parameters H, R). However, it is well known that under stress the soil medium does not behave as "Hooke's body" and hence the curve representing the soil settlement with time obtained from equations (1) is not consistent with the empirical results. Biot's model generates too great instantaneous settlements (just after the imposing of load), whereas the values observed in the laboratory increase gradually. In other words: in an edometrical experiment, the surface of the soil sample under load does not "deflect" instantaneously from a considerable value, but settles with time at a certain velocity, in the direction consistent with the direction of acting load. Therefore it can be assumed that this motion is a manifestation of the viscosity of the soil medium. The introduction of the rheological element, with the property of viscosity (dash-pot in figure 1), to the classical Biot's model can result in (depending on the configuration) the suppression of the instantaneous settlements. Such an action makes model solutions considerably closer to the real behaviour of the medium.

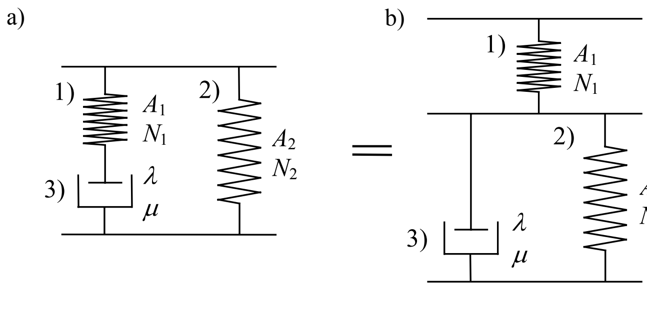

Fig. 1. Zener's body as the composition of: a) Maxwell's body 1) + 3) and Hooke's body 2); b) Hooke's body 1) and Kelvin's body 2) + 3)

Theoretically there are an infinite number of rheological models that can correctly describe various soil properties. Nevertheless, according to the opinion of the authors cited, the choice of an adequate model is always a compromise between the accuracy of the description of the soil properties under examination on the one hand and the possibility of determining the parameters of rheological elements and calculation possibilities on the other one.

In the search for a rheological model used to attain the objective specified in the introduction, the Zener's body was adopted (figure 1a and b). It consists of two springs (1), 2)) and a dash-pot (3)), where the configurations a) and b) of rheological elements are equivalent.

According to KISIEL and LYSIK [6] the most versatile in describing the general properties of soil is simple Tan's model, which is actually Zener's model incorporated into Terzaghi's column. The versatility originates from the fact that after removing spring 1) we obtain Kelvin's body, after removing spring 2) we obtain Maxwell's body, whereas after removing elements 2) and 3) we obtain Hooke's body.

It seems possible to interpret the particular elements of Zener's body by referring to going through the whole consolidation process. And hence, spring 1) represents "structural elasticity". It relates to the situation where under load the soil pores are closing, the water is squeezed out and the particles making up the skeleton change configuration. Then the load is being transferred onto the skeleton grains which deform elastically – this is represented by spring 2). When the whole load is transferred onto the solid phase, a long-lasting process of slow viscous flow of the medium (creeping) begins. This can be regarded as the "sliding" of grains on the layers of the water film bound to their surfaces, which is represented by dash-pot 3). As shown in figure 1, each rheological element is characterized by two parameters. A represents the bulk modulus of the soil, N is the shear modulus, whereas λ and μ stand for the volumetric and dynamic viscosity of the skeleton, respectively. The stress–strain relationship for Zener's body that takes into account the stress transferred by the fluid is denoted by:

$$\sigma_{ij} = 2N_2\psi_p\varepsilon_{ij} + A_2\psi_o\varepsilon\delta_{ij} + \frac{Q}{R}\sigma\delta_{ij} - \frac{Q^2}{R}\varepsilon\delta_{ij}, \tag{2}$$

where:

$$\psi_p = \frac{1+\Gamma\cdot T\cdot s}{1+T\cdot s}, \quad \psi_o = \frac{1+\Gamma_1\cdot T_1\cdot s}{1+T_1\cdot s},$$

$$\Gamma = \frac{N_1+N_2}{N_2}, \quad \Gamma_1 = \frac{A_1+A_2}{A_2},$$

$$T = \frac{\mu}{N_1}, \quad T_1 = \frac{\lambda}{A_1},$$

whereas s is Mikusinski's operator, $s = \partial/\partial t$.

Introducing the operators ψ_p and ψ_o to relation (1), one-dimensional equation of Biot consolidation with Zener's rheological skeleton is expressed by:

$$\begin{cases} N_2\psi_p \dfrac{\partial^2 u_1}{\partial x_1^2} + \left(N_2\psi_p + A_2\psi_o - \dfrac{Q^2}{R} \right) \dfrac{\partial \varepsilon}{\partial x_1} = -\dfrac{H}{R} \dfrac{\partial \sigma}{\partial x_1}, \\ \dfrac{k}{f^2} \nabla^2 \sigma = \dfrac{1}{R} \dot{\sigma} - \dfrac{H}{R} \dot{\varepsilon}. \end{cases} \tag{3}$$

As the solution of the equation set shown above, in the case where the relaxation time $T \neq T_1$, proves difficult (the second time derivative appears), the calculations were performed assuming that $T = T_1$. The other terms are adopted as follows:

- $N_1 = 10^7$ [Pa], $A_1 = 5 \cdot 10^7$ [Pa],
- $N_2 = 10^7$ [Pa], $A_2 = 5 \cdot 10^7$ [Pa],
- $\mu = 10^{11}$ [Pa·s], $T = T_1 = \mu / N_1 = \lambda / A_1 = 10^4$ [s],
- $k = 10^{-11}$ m/s, $f = 0.35$,
- the length of the soil sample (region of consolidation) is 10 m.

It has to be stressed that the rheological values mentioned above are approximate – established from the literature (EMMRICH [3]).

The boundary value problems given and their solutions are presented in the next, i.e., the third section.

3. THE BOUNDARY VALUE PROBLEMS AND SOLUTIONS

The problem of soil medium consolidation was solved using the finite element method implemented in code FlexPDE5. Non-dimensional values were used for the calculations and the reference values were assumed as follows: the displacement of medium $U_0 = 10^{-2}$ m, the fluid stress $S_0 = 10^5$ Pa. The geometry of the region and the boundary conditions given are shown in figure 2.

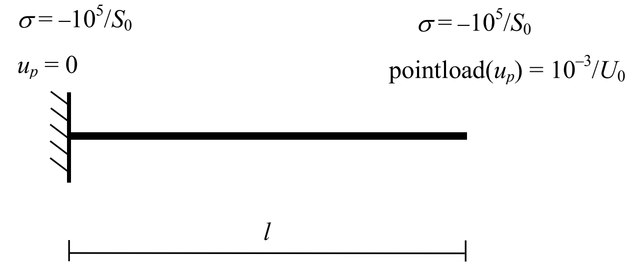

Fig. 2. Geometry of the consolidation region and boundary conditions

Firstly, the problem of Biot's body consolidation was solved. In this case only spring 2) is active (figure 1) and filtration occurs (equation (1)). Hence, it can be assumed that the settlements (displacements) obtained correspond to maximum values

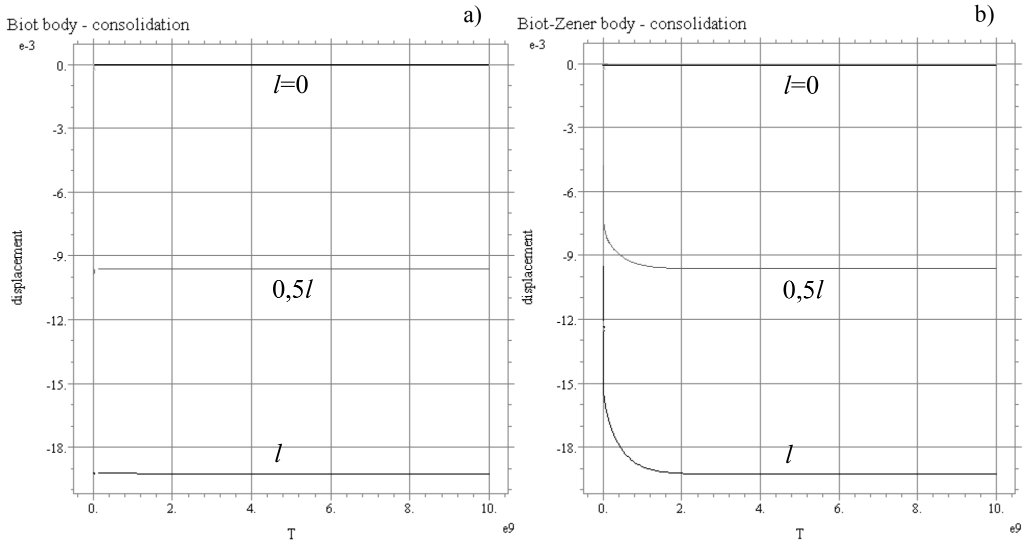

Fig. 3. The settlement values obtained from models: a) Biot's ($t = 10^9$ s), b) Zener's ($t = 10^9$ s)

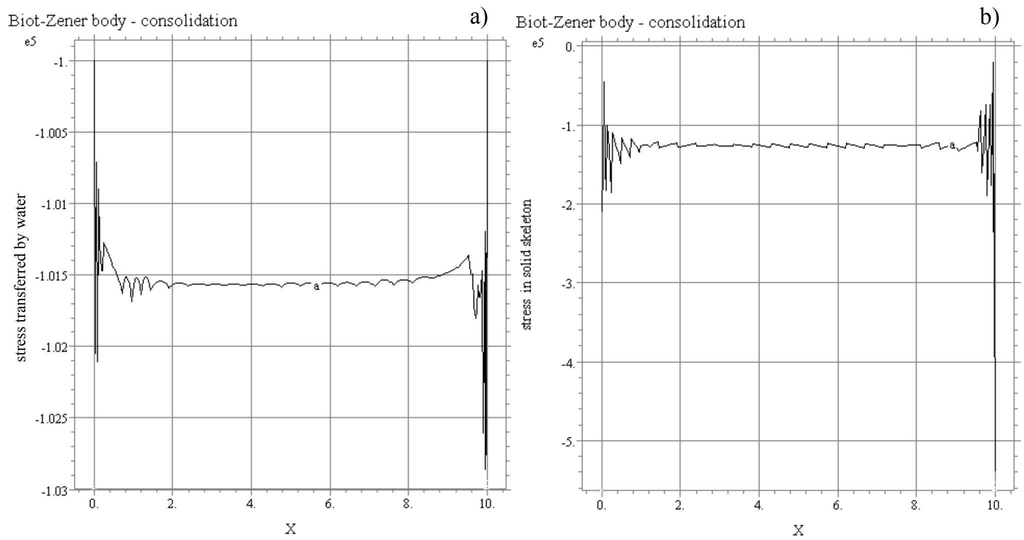

Fig. 4. The distribution of the stress transferred by: a) water, b) soil skeleton after the time $t = 10^3$ s

of instantaneous settlements (figure 3a). (In the example shown, instantaneous settlements are nearly identical to the final settlements).

In the second step, however, in Zener's model, the dash-pot 3) was disconnected and the problem of Biot's body consolidation was solved again. This time with springs 1), 2) and filtration (equation (1)). This situation corresponds to the minimum value of instantaneous settlement, in this case the value obtained was $-9.6e^{-3}$ m.

In the two cases described above, creeping of the medium appears, apart from the instantaneous settlements mentioned. However, this time it results from the viscous resistance of the filtrating fluid.

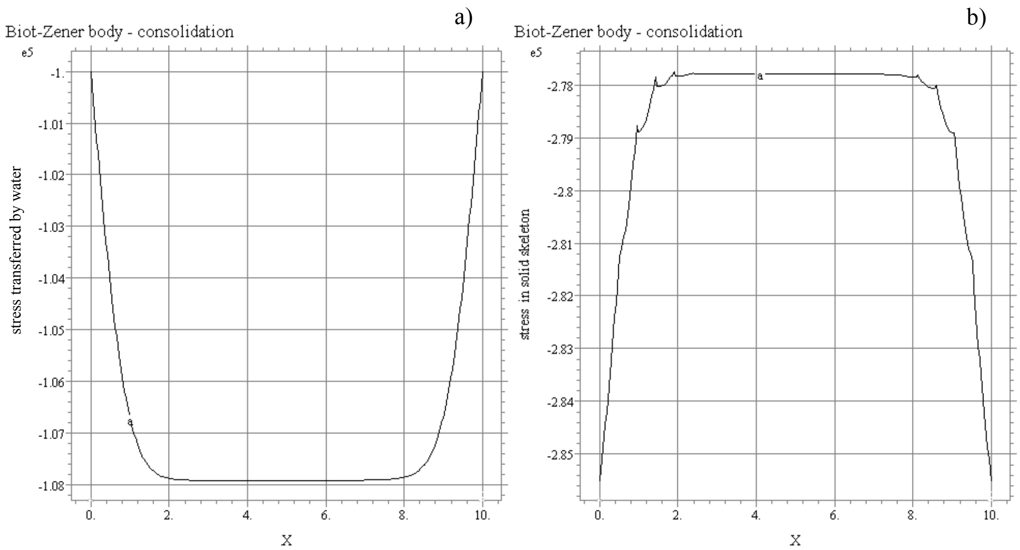

Fig. 5. The distribution of the stress transferred by: a) water, b) soil skeleton after the time $t = 10^7$ s

Then, the problem of the consolidation of the medium with Zener's skeleton was solved. In this case, the boundary value problem and an initial condition complemented each other. The initial condition was as follows: the fluid stress $\sigma\,(t = 0) = -10^5/S_0$ and displacement $u_p(t = 0) = -9.6e\text{-}3 \cdot x/U_0/l$, where the variable x stands for the point's instant coordinate. The resulting values of settlements are shown in figure 3b. It can be seen that the maximum values, from the classical Biot's model and from the model that considers rheological properties of the skeleton, agree.

The process of consolidation was observed at the times: $t = 0.2$ s, $t = 1$ s, $t = 10$ s, $t = 100$ s, $t = 1000$ s, $t = 10^4$ s ... $t = 10^{11}$ s, but only some results are presented in this paper.

The distributions of the stress transferred by water and the soil skeleton after the times $t = 10^3$ s and $t = 10^7$ s are shown in figures 4 and 5, respectively.

4. DISCUSSION OF THE RESULTS AND CONCLUSIONS

The results of the numerical analyses presented in this article seem to agree with the engineering intuition. Even by analyzing the pressure distributions in the soil medium and in the fluid, we observe that during the first stages of consolidation ($t = 10^3$ s) the stress in the fluid is similar to that in the soil skeleton, whereas at the end of this process ($t = 10^7$ s) the stress in the skeleton is much greater. The maximum settlements obtained in both cases, i.e., for the classical Biot's model and for Zener's body, take the same values. There is an essential difference between the instantaneous settlements, which in the case of the rheological model being evaluated are smaller. This statement, although expected, allows an answer to the question about the influence of "viscosity" of the soil skeleton on the stress–strain process.

It should be emphasized that in the numerical experiment with Zener's body, the adoption of adequate initial conditions remains crucial.

Taking account of the fact that informatics and computer technology development provide better and better computational tools, it seems advisable to search for models and solutions that most accurately describe the natural processes. In this context, the present paper is only a minor step in distinguishing which of the simple rheological models describes adequately the general properties of the soil medium.

REFERENCES

[1] BIOT M.A., *General theory of three-dimensional consolidation*, Journal of Applied Physics, February 1941, Vol. 12, 155–164.

[2] DERSKI W., *Zarys mechaniki ośrodków ciągłych*, PWN, Warszawa, 1975.

[3] EMMRICH R., *Experimental verification of electro-kinetic consolidation model*, PhD dissertation in Polish, Wrocław University of Technology, 1984, Report ser. PRE, No. 307.

[4] KISIEL I., *Reologia w budownictwie*, Arkady, Warszawa, 1967.

[5] KISIEL I. (ed.), DERSKI W., IZBICKI R., MRÓZ Z., *Mechanika skał i gruntów*, PWN, Warszawa, 1982.

[6] KISIEL I., LYSIK B., *Zarys reologii gruntów. Działanie obciążenia statycznego na grunt*, Arkady, Warszawa, 1966.

[7] SAWICKI A., *Mechanika kontinuum. Wprowadzenie*, Wydawnictwo IBW PAN, Gdańsk, 1994.

[8] STRZELECKI T. (ed.), KOSTECKI S., ŻAK S., *Modelowanie przepływów przez ośrodki porowate*, DWE, Wrocław, 2007.

[9] FlexPDE5 v5.0.7, "FlexPDE Reference", www//http.flexPDE.com

[10] HUANG T.H., CHANG C.S., CHAO C.Y., *Experimental and mathematical modeling for fracture of rock joint with regular asperities*, Engineering Fracture Mechanics, 2002, Vol. 9, 1977–1996.

[11] LEE S.D., HARRISON J.P., *Empirical parameters for non-linear fracture stiffness from numerical experiments of fracture closure*, International Journal of Rock Mechanics and Mining Sciences, 2001, 38, 721–727.

[12] LONDE P., SABARLY F., *La distribution des perméabilités dans la fondation des barrages voûtés en fonction du champ de contrainte*, Proc. 1st Congr. Rock Mechanics, 25 Septembre–1 Octobre 1966, Lisbon, Lab Nac Eng Civil, Vol. II, 517–522.

[13] NAZRIDOUST K., AHMADI G., SMITH D.H., *A new friction factor correlation for laminar single-phase flows through rock fractures*, Journal of Hydrology, 2006, Vol. 329, 315–328.

[14] QIAN J., ZHAN H., ZHAO W., SUN F., *Experimental study of turbulent unconfined groundwater flow in a single fracture*, Journal of Hydrology, 15 September 2005, Vol. 311, Issues 1–4, 134–142.

[15] RUTQVIST J., STAPHANSSON O., *The role of hydromechanical coupling in fractured rock engineering*, Hydrology Journal, 2003, 11, Springer-Verlag, 7–40.

[16] WITHERSPOON P.A., WANG J.S.Y., IWAI K., GALE J.E., *Validity of cubic law for fluid flow in a deformable rock fracture*, Water Resour. Res., 1980, 16, 1016–1024.