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# DETERMINATION OF PERMEABILITY COEFFICIENT OF SATURATED CLAY BASED ON LINEAR SEGMENT OF SETTLEMENT CURVE

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**Abstract:** In this article, the conception of determination of permeability coefficient $k$ based on linear segment of the settlement curve in a compression test is presented. By observing the evolution of the settlement curve of a soil, it is possible to find in its initial part a linear relation between the settlement and time. For clays, the time interval ranges from about 45 sec to a few minutes, as measured from the moment a load has been imposed. The linear character of the graph means that rheological effects are absent and that settlement velocity is constant and equal to percolation velocity. Thus, it may be supposed that within this time interval the permeability coefficient is approximately constant, too. Hence, the value of $k$ calculated on the basis of this linear part of the settlement curve should be reliable.

In this paper, the values of permeability coefficient $k$ for clay, obtained in triaxial apparatus were compared with those obtained in oedometer test (based on the linear segment of the settlement curve). The results obtained appear to be promising. Nevertheless, since the number of tests carried out so far is rather small, it is necessary to perform more experiments to verify our conception.

## 1. INTRODUCTION

The permeability coefficient known as "$k$" from Darcy's law ($v = -k \cdot i$) is generally considered as a fundamental parameter describing the permeability properties of mineral sealing materials. These materials, which contain significant amounts of clay particles – usually above 20% [15], are often used as sealing elements in waste landfills, earthen dams, embankments. They can also be used as impermeable screens in structures such as underground water reservoirs, containers of chemical or radioactive materials.

Considering the above, it is not surprising that determining the value of the permeability coefficient for these materials is of interest to so many researchers. A basic problem consists in determining a reliable value for parameter $k$ in fine grained materials, with $k$ values below $10^{-9}$ m/s [8], [13]–[15]. The evaluation of hydraulic conductivity has encountered a number of problems resulting from the nature of permeability of fine grained soils which depends on mineralogical composition, microstructure, initial state of saturation, chemical composition of percolating water, initial hydraulic gradient or drainage conditions [13].

Depending on the research capacity and the anticipated uses of the material, a variety of methods can be employed to determine hydraulic conductivity. They can generally be divided into *in situ* methods, laboratory methods and mathematical models.

Importantly, each of the methods is subject to some measurement errors, which have their source in the assumptions adopted, the ways of determining the indirect values and applied technical solutions.

Many authors have described various methods and testing equipment and provided examples of the sources of errors of in situ experiments [6], laboratory tests in which the determination of the permeability coefficient was based on the consolidation curve [8], [13], laboratory tests in a triaxial apparatus or prototype apparatus chamber [4], [8], [11], [13]. Descriptions of some analytical models, e.g. Kozeny–Carman, Olsen's cluster model, which takes into account the impact of electrical double layer, and a model using neural networks and the scope of their applicability can also be found [1], [2], [10], [14].

Due to the already highlighted characteristics of the filtration process in low permeable soils, and numerous sources of measurement errors, it is difficult to identify the most versatile and at the same time, the most reliable method of determining the value of permeability coefficient. The intention of the authors of this article was not to resolve this matter, but to focus on the search for a possible method, rather simple, universal, not requiring long-term measurements, which simultaneously would provide sufficiently reliable results for engineering applications.

After an initial analysis of the results it appears that a method of determining the permeability coefficient based on linear segment of consolidation curve can meet our expectations.

In section 2, the concept of establishing the permeability coefficient $k$ based on the oedometric consolidation curve is presented. Section 3 contains a brief description of the soil tested, a description of laboratory tests and the results obtained from the triaxial apparatus and oedometer. Section 4 demonstrates how to develop the results (determination of $k$) from oedometric tests. In section 5, the results are discussed and conclusions presented.

## 2. THE CONCEPT OF ESTABLISHING THE PERMEABILITY COEFFICIENT BASED ON THE LINEAR PART OF THE CONSOLIDATION CURVE

The entire consolidation process can be divided into two successive stages: preliminary and secondary consolidation [12], [16]. It is generally accepted that the consolidation process begins with the application of the load to a fully saturated soil sample [16]. Then, there is almost immediate compression of the air bubbles and its dissolution in pore water, which corresponds to the deformation of the sample observed in the first few seconds of the consolidation process. Next, the water in which the previously entrapped air has dissolved is partially compressed. Finally, at a specified load, porous water becomes incompressible and begins to be "extruded" from the voids – filtration process starts.

From this moment, the settlement of the soil sample depends only on the amount of water squeezed out, hence the settlement velocity is equal to the "squeezing" velocity of pore water, that is, the percolation velocity.

At this stage of the consolidation, the consolidation curve is linear.

As Zhikhovich pointed out [16], the consolidation described above, proceeding in a short period of time (measured from the application of the load) is devoid of the impact of rheological processes, which have not yet had time to develop.

Finally, the soil particles begin to undergo re-orientation (rotate), which is a manifestation of the developing rheological processes.

The pore water pressure in the soil sample decreases, resulting in a decrease of percolation velocity. The settlement curve related to the time loses its linear character, the preliminary consolidation ends and the secondary consolidation begins.

It is apparent from the foregoing description that the process of the settlement of the sample is linear only for a short period of time, soon after the consolidation process has begun.

Since the percolation velocity (the velocity of the outflow of water from the sample) determines the velocity of the settlement of the sample, the linearity of this process means that the percolation velocity is constant. Therefore, in that time period the permeability coefficient can be roughly assumed to be constant.

Given the above considerations, and being limited only to the phase of preliminary consolidation, and more specifically to the linear relation between settling and time, we attempt to determine the value of permeability coefficient based on Terzaghi's one-dimensional consolidation theory.

It seems that posing the problem in such a way makes the following assumptions of the theory of consolidation less disputable:

a) the strains are small and one-dimensional,

b) the soil is saturated,

c) the soil grains and the pore fluid are incompressible,

d) the soil is homogeneous,

e) the compressibility and permeability of the soil remain constant during the consolidation process,

f) the flow is one-dimensional and Darcy's law is valid,

g) there is a linear relationship between the effective stress and strain,

h) rheological processes do not occur in the soil.

One-dimensional consolidation equation can be presented in the form:

$$\frac{\partial u}{\partial t} = C_{\mathrm{v}} \frac{\partial^2 u}{\partial z^2}, \tag{1}$$

where:

$u$ – pore water pressure,

$c_{\mathrm{v}}$ – consolidation coefficient:

$$C_v = \frac{kM_0}{\gamma_w}, \tag{2}$$

where:

$k$ – coefficient of permeability,

$M_0$ – oedometric modulus of primary compression,

$\gamma_w$ – bulk weight of water.

The solution of equation (1) is usually presented in the form: $U = f(T_v)$, where $U$ is the consolidation degree (understood as the ratio between settlement after a time period to the total settlement), and $T_v$ is a time factor to be set by the formula:

$$T_v = \frac{c_v t}{H^2}. \tag{3}$$

In this equation, $t$ is the time after which the degree of consolidation $U$ is achieved, and $H$ is the length of the filtration path (usually equal half the height of the soil sample).

The average value of the degree of consolidation can be determined from the following formula, which is the solution to equation (1) in the form of Fourier series:

$$U = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} e^{(-M^2 T_v)} = f(T_v) \tag{4}$$

where $M = \frac{\pi}{2}(2m + 1)$.

As mentioned earlier, the concept presented here assumes that the value of parameter $k$ should be determined on the basis of the linear part of the settlement graph of the sample. Thus, identifying this linear segment is the starting point for further calculations. A detailed procedure for determining $k$ values is presented in section 4.

## 3. SOIL PROPERTIES AND TESTING METHODS

To verify the ideas presented in the previous section, laboratory tests in triaxial apparatus TRX and oedometer OED were performed. Permeability coefficient $k$ obtained by the TRX tests was used as the reference value in relation to the results obtained on the basis of the consolidation curve.

The clay from the upper level of Poznań Series clays, from Brzeg Dolny (Wrocław region) was chosen for testing. In order to preserve the intact structure of soil, the soil sample was manually cut out, in the form of blocks of 20×20×30 cm in size, from the slope of the excavation. They were taken from the depth of 4.5 m below soil surface.

The soil blocks in the laboratory were cut into smaller cubes and then pressed into samplers to obtain samples for testing in triaxial apparatus TRX and oedometer OED. The diameters of the samplers were 36 and 65 mm, respectively.

The oedometer ring was covered with silicone paste to reduce frictional resistance during the test, and thus the contact between the soil and the oedometer's ring was sealed [7], [9].

Basic physical properties of the soil tested are summarized in Table 1. The soil was semi-stiff clay, with the water content of 20.3% and bulk density of 2.04 g/cm$^3$. The degree of saturation value varies within the range of 0.95 and indicates the unsaturated state of soil in the presence of a continuous phase of water [5].

The tests have shown that the granulometric composition of the soil is 70.2% clay and 29.8% silt, with no sand fraction being found. The mineral composition (given in [3]) was predominated by kaolinite; smectite and illite were also present. The composition of ion exchange complex was dominated by $Ca^{2+}$.

Table 1

Basic physical properties of tested soil

| Physical properties of soil | | | Minearological composition 1)<br>Chemical composition [%] 1) | |
|---|---|---|---|---|
| Natural water content | $w_n$ | 20.3% | $SiO_2$ | 56.02 |
| Bulk density | $\rho$ | 2.04 g/cm$^3$ | $AL_2O_3$ | 18.08 |
| Density | $\rho_s$ | 2.66 g/cm$^3$ | FeO | 0.34 |
| Coefficient of porosity | $e$ | 0.57 | $Fe_2O_3$ | 10.0 |
| Plasticity limits | $w_p$ | 28.0% | CaO | 0.86 |
| Liquidity limits | $w_L$ | 70.4% | Smektyt | ++ |
| Plasticity degree | $I_L$ | –0.18 | Kaolinit | +++ |
| Plasticity index | $I_P$ | 42.4% | Illit | ++ |
| Saturation degree | $S_r$ | 0.95 | Ion exchange complex 1) | |
| Clay | Cl | 70.2 | $Ca^{2+}$ | 10.0 |
| Silt | Si | 29.8 | $Mg^{2+}$ | 4.5 |
| Sand | Sa | 0 | $K^+$ | 0.5 |
| | | | $Na^+$ | 0.78 |
| | | | CEC 1) | 16,30 |
| | | | specific surface S 1) | $171\times10^3$ m$^2$/kg |

1) values after [3]

Permeability coefficient $k$ was determined directly and undirectly in the laboratory tests.

Directly, the value of coefficient $k$ was determined in the triaxial apparatus TRX. The test was conducted by a standard method as described in DIN [17]. The triaxial apparatus ELE Int. equipped with electronic pressure and volume changes transducers, enabled very accurate measurements. Soil sample with a diameter of 38 mm and height of 76 mm was initially saturated with water until the coefficient *b* from Skempton's equation became equal to 0.97 [7]. This process took about 4 weeks.

Then filtration was initiated by increasing the back pressure of water at the base of the sample. Filtration took place from the bottom towards the top of the soil so as to allow air bubbles to escape from the pores of the soil. During the test the measurements of the water volume flowing in and out of the sample soil were done.

The study was conducted for three different values of effective stress 150, 200 and 250 kPa and the corresponding hydraulic gradient $i$ = 200, 263, 328. Filtration tests were carried out by using distilled water at 20 ± 1 °C. The study was conducted with high hydraulic gradients to increase velocity of flowing water to reduce reading error of the measurement of the volume of flowing water. The high hydraulic gradients also shortened the testing time.

For the interpretation of the results it was assumed that the sample was completely saturated with water. The measurement errors in the TRX were not taken into account [8], [13], [14]. To eliminate some errors in the determination of value of coefficient $k$ from TRX test, the measurements were done after 48 h from the filtration process had been initiated. The filtration was observed during the following 24h. The obtained values of the parameter $k$ are given in Table 2.

Table 2

The permeability coefficient $k$ values from TRX test
($\sigma'_3$ – cell pressure, $i$ – hydraulic gradient, $k$ – permeability coefficient)

| $\sigma'_3$ [kPa] | $i$ [–] | $k$ [m/s] |
|---|---|---|
| 150 | 200 | $7.5\ 10^{-12}$ |
| 200 | 263 | $3.3\ 10^{-12}$ |
| 300 | 328 | $2.2\ 10^{-12}$ |

Indirectly, the value of $k$ was determined based on the results from standard oedometer test OED using an incremental load procedure, IL [7].

The test started from the initial flooding of the sample with distilled water to saturate the sample. Due to the swelling observed after the flooding it was necessary to gradually increase the load so as to avoid changes in the volume of soil being tested.

The swelling pressure was balanced by the load equal to 116.8 kPa. Then, the test began by gradually applying the appropriate load. Each subsequent step load $\sigma_{vi}$ was two times greater than the previous one $\sigma_{vi-1}$ ($\sigma_{vi}/\sigma_{v\ i-1} = 2$). Successive load was applied when the change of the soil sample height was less than 0.002 mm within 24 hours.

The sample was increasingly loaded with the load up to 1 MPa. The value of the coefficient $k$ was determined from the linear part of the consolidation curves plotted for each load (figures 1–5) with the method described in section 4.

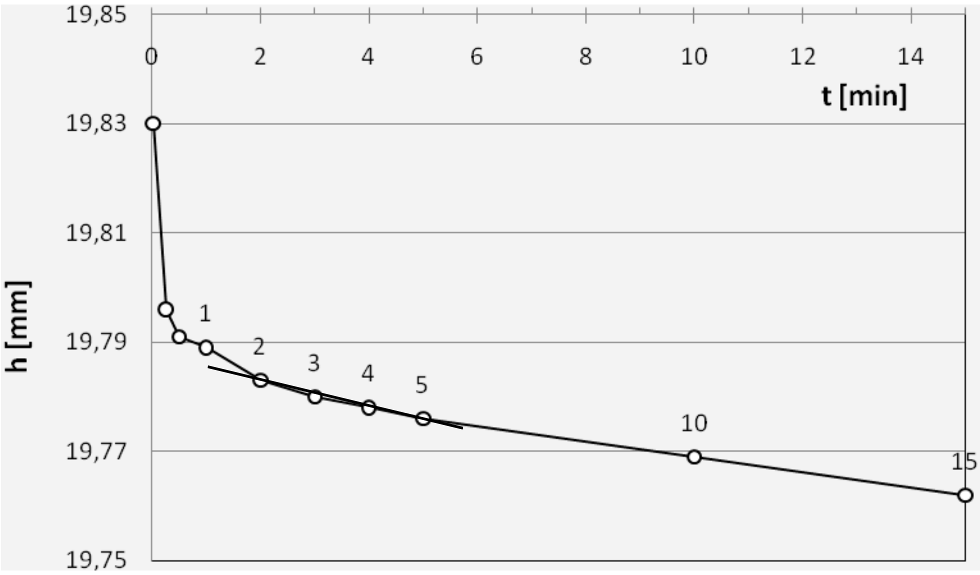


Fig. 1. Evolution of settlement versus time under normal stress $\sigma_{vi}$ = 210.411 kPa

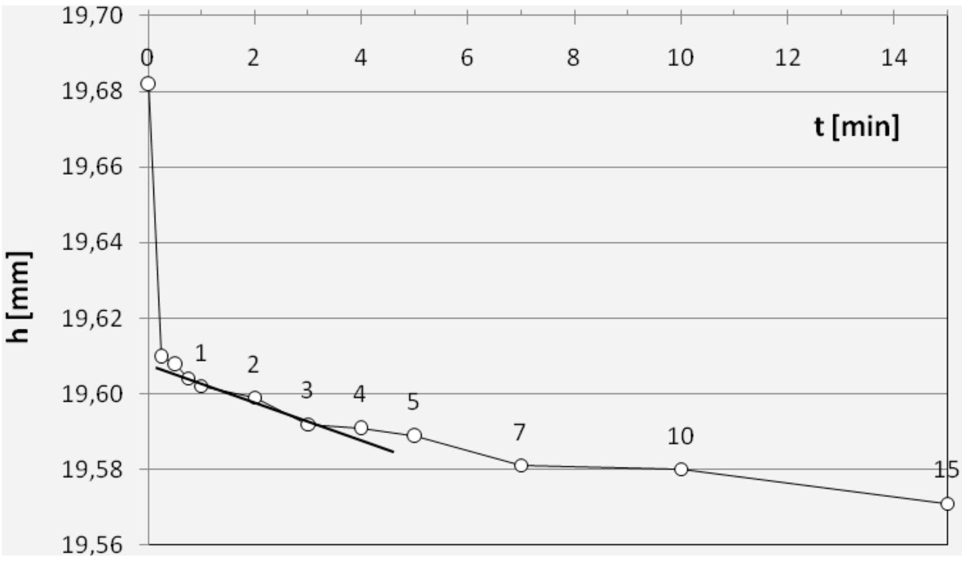


Fig. 2. Evolution of settlement versus time under normal stress $\sigma_{vi}$ = 303.674 kPa

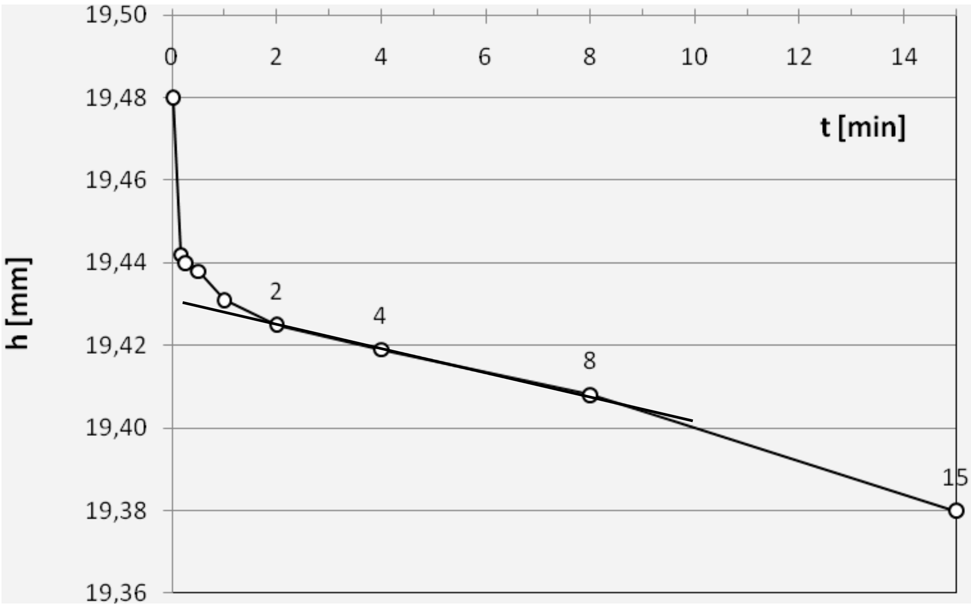


Fig. 3. Evolution of settlement versus time under normal stress $\sigma_{vi}$ = 443.050 kPa

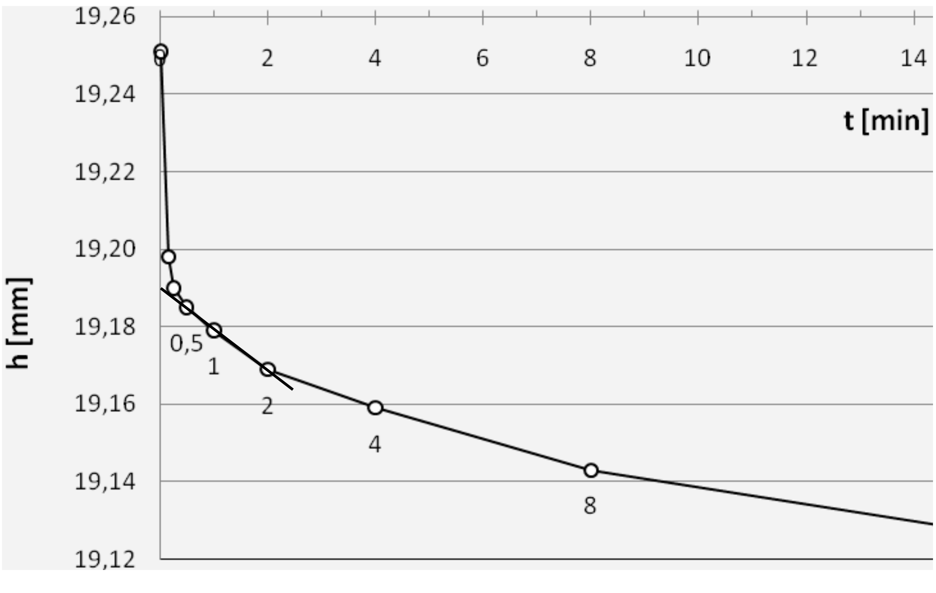


Fig. 4. Evolution of settlement versus time under normal stress $\sigma_{vi}$ = 675.623 kPa

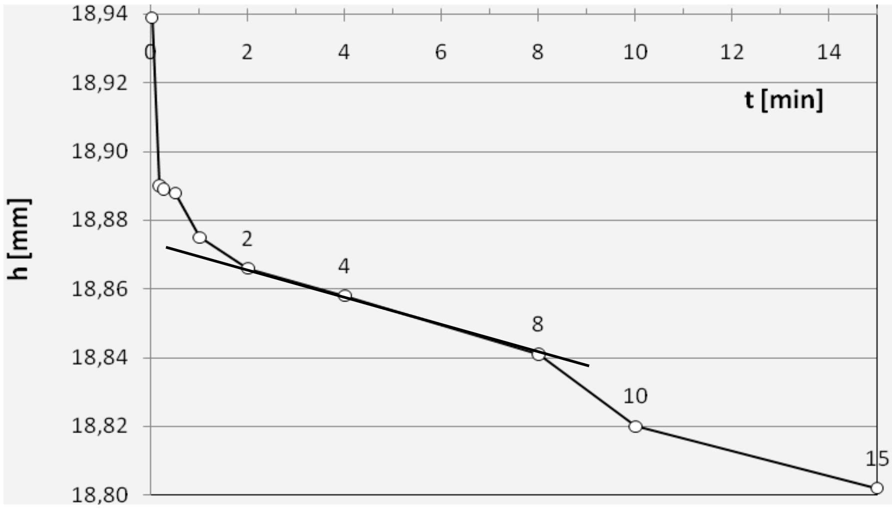


Fig. 5. Evolution of settlement versus time under normal stress $\sigma_{vi}$ = 1001.738 kPa

## 4. DETERMINATION OF THE PERMEABILITY COEFFICIENT BASED ON THE OEDOMETRIC CONSOLIDATION CURVE

After completion of the oedometric tests (described in Section 3) and plotting the graphs (figures 1–5) periods (for the subsequent load steps) indicating a linear process of settlement over time were identified. According to [16] in the case of clay, the linear part of the consolidation curve must be sought not earlier than after 45 s from the beginning of the test and its end after 2–3 minutes. In individual cases, this period may be extended. Zhikhovicz [16] in his study obtained a linear segment of the curve, which began in 2 min and ended 5 min after the consolidation had started.

In our case, the choice of the length of the period for which the permeability coefficients were determined was dependent on two factors:

i) the similar slope of adjacent sections of consolidation curve, and

ii) the time of consolidation, which is a priori no more than a few minutes.

Then the value of sample settlement was calculated: $\Delta h = h(t_p) - h(t_k)$, where $h(t_p)$ and $h(t_k)$ are the heights of the sample at the beginning and at the end of the preliminary consolidation, respectively, whose time scope was limited by the adopted period. In other words, $h(t_p)$ and $h(t_k)$ are treated as the initial height of the sample and its height after the end of the consolidation.

In the next step, using an analogy to the Casagrande method, from formula (4) the time factor $T_v$ corresponding to the degree of consolidation $U$ = 0.5 was determined (the calculations were done for $m$ = 100).

Other parameters were defined as follows: time $t_{50}$, after which there is a 50% of consolidation, is $(t_k - t_p)/2$, and the average path length of filtration $H = [(h(t_p) + h(t_k))/2]/2$. Oedometric modulus of primary compression was calculated based on the linear section of the consolidation curve. Finally, combining formulas (2) and (3) the permeability coefficient $k$ was determined:

$$k = \frac{T_v \gamma_w H^2}{M_0 t_{50}}. \qquad (5)$$

The values of the permeability coefficient obtained for OED-IL test (from figures 1–5) and calculated applying the procedure presented, are shown below:

$\sigma_{vi}$ = 210.411 kPa

| $h(t_p = 2)$ | $h(t_k = 5)$ | $\Delta h$ | $U_{50}$ | $T_v$ | $\gamma_w$ | $H$ | $M_0$ | $t_{50}$ | $k_{OED}$ |
|---|---|---|---|---|---|---|---|---|---|
| [mm] | [mm] | [mm] | [–] | [–] | [N/m$^3$] | [mm] | [Pa] | [s] | [m/s] |
| 19.783 | 19.776 | 0.007 | 0.5 | 0.197 | 9793 | 9.8898 | 2.0819E+08 | 270 | **3.5E-12** |

$\sigma_{vi}$ = 303.674 kPa

| $h(t_p = 0.75)$ | $h(t_k = 3)$ | $\Delta h$ | $U_{50}$ | $T_v$ | $\gamma_w$ | $H$ | $M_0$ | $t_{50}$ | $k_{OED}$ |
|---|---|---|---|---|---|---|---|---|---|
| [mm] | [mm] | [mm] | [–] | [–] | [N/m$^3$] | [mm] | [Pa] | [s] | [m/s] |
| 19.604 | 19.592 | 0.012 | 0.5 | 0.197 | 9793 | 9.7990 | 4.9610E+08 | 67.5 | **5.5E-12** |

$\sigma_{vi}$ = 443.050 kPa

| $h(t_p = 2)$ | $h(t_k = 8)$ | $\Delta h$ | $U_{50}$ | $T_v$ | $\gamma_w$ | $H$ | $M_0$ | $t_{50}$ | $k_{OED}$ |
|---|---|---|---|---|---|---|---|---|---|
| [mm] | [mm] | [mm] | [–] | [–] | [N/m$^3$] | [mm] | [Pa] | [s] | [m/s] |
| 19.425 | 19.408 | 0.017 | 0.5 | 0.197 | 9793 | 9.7082 | 5.0625E+08 | 180 | **2.0E-12** |

$\sigma_{vi}$ = 675.623 kPa

| $h(t_p = 0.5)$ | $h(t_k = 2)$ | $\Delta h$ | $U_{50}$ | $T_v$ | $\gamma_w$ | $H$ | $M_0$ | $t_{50}$ | $k_{OED}$ |
|---|---|---|---|---|---|---|---|---|---|
| [mm] | [mm] | [mm] | [–] | [–] | [N/m$^3$] | [mm] | [Pa] | [s] | [m/s] |
| 19.185 | 19.169 | 0.016 | 0.5 | 0.197 | 9793 | 9.5885 | 8.1011E+08 | 45 | **4.9E-12** |

$\sigma_{vi}$ = 1001.738 kPa

| $h(t_p = 2)$ | $h(t_k = 8)$ | $\Delta h$ | $U_{50}$ | $T_v$ | $\gamma_w$ | $H$ | $M_0$ | $t_{50}$ | $k_{OED}$ |
|---|---|---|---|---|---|---|---|---|---|
| [mm] | [mm] | [mm] | [–] | [–] | [N/m$^3$] | [mm] | [Pa] | [s] | [m/s] |
| 18.866 | 18.841 | 0.025 | 0.5 | 0.197 | 9793 | 9.4268 | 7.5595E+08 | 180 | **1.3E-12** |

## 5. DISCUSSION

There is a certain accordance of the results when comparing the values of the coefficient $k$ obtained on the basis of TRX and OED. In TRX tests for $\sigma'_3 = 200$ kPa $k$ was found to equal $k = 3.3 \cdot 10^{-12}$ m/s, and in OED tests for $\sigma = 210$ kPa value of $k$ was $k = 3.5 \cdot 10^{-12}$ m/s. By contrast, there is a 2.5 times difference in $k$ values obtained at a stress equal to about 300 kPa. This result does not necessarily disqualify the method of calculation presented. Comparing the $k$ values obtained from OED under load of 210, 443 and 1001 kPa we observed a reduction of permeability coefficient, which is consistent with our expectations.

The primary difficulty connected with the method presented lies in identification the section of consolidation curve representing the linear relationship of settlement in time. Observing the graphs of consolidation it seems that the cause of these difficulties is most likely too small a number of sampling points (too large time-intervals between consecutive measurements). Measuring the settlement of the sample at intervals of 10 or maybe 15 seconds should make it easier to find a linear relationship $h(t)$ and thus improve the "quality" of coefficient $k$ being determined.

At the moment, it is impossible to categorically formulate conclusions on the advantages and disadvantages of the approach presented. Too few tests were performed, and thus there is a lack of verification of the method. At this stage, the authors of this article consider it rather an indication of an additional possibility of indirect determination of coefficient $k$. However, in our opinion the method presented here seems to be promising because of its simplicity, relatively short time of testing, low cost of testing equipment and the fact that the results do not differ much (which should be still confirmed) from those obtained using the method of testing in triaxial apparatus, which is generally accepted as the most accurate one [13], [14].

## REFERENCES

[1] ACHARI G., JOSHI R.C., BENTLEY L.R., CHATTERJI S., *Prediction of the hydraulic conductivity of clays using the electric double layer theory*, Can. Geotech. J., Vol. 36(5), 1999, 783–792.

[2] ANANDARAJAH A., *Mechanism Controlling Permeability Change in Clays due to Changes in Pore Fluid*, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 129, No. 2, 2003, 163–172.

[3] CHOMA-MORYL K., *Zmienność własności fizycznych iłów poznańskich okolic Wrocławia na tle ich genezy i litostratygrafii*, Geol. Sud., Vol. 23, No. 1, 1988, 1–57.

[4] DIXON D.A., GRAHAM J., GRAY M.N., *Hydraulic conductivity of clays in confined tests under low hydraulic gradients*, Can. Geotech. J., Vol. 36(5), 1999, 815–825.

[5] FREDLUND D.G., RAHARDJO H., *Soil Mechanics for Unsaturated Soils*, New York: John Wiley&Sons, Inc., 1993.

[6] HIRD C.C., SRISAKTHIVEL S., *Laboratory investigation of permeability measurement in clay using outflow from unsupported cavities*, Geotechnique, Vol.55, No. 5, 2005, 393–402.

[7] HEAD K.H., *Manual of soil laboratory testing*, London: Pentech Press, Vol. 1–3, London, 1992.

[8] JAROMIŃSKA M., DEMBICKI E., *Określenie współczynnika filtracji gruntów słabo przepuszczalnych*, Inżynieria Morska i Geotechnika, nr 5, 1999.
[9] LEROUEIL S., *Compressibility of clays: Fundamental and practical aspects*, J. Geotech. Eng., ASCE, 6, 1996.
[10] NAJJAR Y.M., BASHEER I.A., *Utilizing computational neural networks for evaluating the permeability of compacted clay liners*, Geotechnical and Geological Engineering, Vol. 14, 1996, 193–212.
[11] SIEMENS G., BLATZ J.A., *Development of a hydraulic conductivity apparatus for bentonite soils*, Can. Geotech. J., Vol. 44, 2007, 997–1005.
[12] STRZELECKI T., KOSTECKI S., ŻAK S., *Modelowanie przepływów przez ośrodki porowate*, Dolnośląskie Wydawnictwo Edukacyjne, Wrocław, 2008.
[13] TAVENAS F., LEBLOND P., JEAN P., LEROUEIL S., *The permeability of natural soft clays*. Part I: *Methods of laboratory measurement*, Can. Geotech. J., Vol. 20, No. 4, 1983, 629–644.
[14] TAVENAS F., LEBLOND P., JEAN P., LEROUEIL S., *The permeability of natural soft clays*. Part II: *Permeability characteristics*, Can. Geotech. J., Vol. 20, No. 4, 1983, 645–660.
[15] WYSOKIŃSKI L., ŁUKASIK S., *Badania szczelności izolacji mineralnych składowisk odpadów*; *Instrukcja 339/96*, Ministerstwo Gospodarki Przestrzennej i Budownictwa, Instytut Techniki Budowlanej, Warszawa, 1996
[16] ZHIKHOVICH V.V., *Determination of the permeability coefficient of saturated clay from the settlement curve in a compression test*, translated from Gidrotekhnicheskoe Stroitel'stvo, No. 12, pp. 36–37, December 1981; Plenum Publishing Corporation, 1982.
[17] DIN 18130-1:1998. *Baugrund, Untersuchung von Bodenproben. Bestimmung des Wasserdurchlassugkeitsbeiwerts*.